ADMISSIBILITY CRITERIA FOR A HIERARCHICAL DESIGN OF HYBRID CONTROL SYSTEMS [1]

Thomas Moor * **Jörg Raisch** † **J.M. Davoren** ‡

* *Research School of Information Sciences and Engineering, Australian National University, Canberra, thomas.moor@anu.edu.au*
† *Lehrstuhl für Systemtheorie technischer Prozesse, Otto-von-Guericke Universität, and Max-Planck-Institut für Dynamik komplexer technischer Systeme, Magdeburg, Germany, raisch@mpi-magdeburg.mpg.de*
‡ *Department of Electrical and Electronic Engineering, University of Melbourne, Melbourne, Australia, davoren@unimelb.edu.au*

Abstract: Many hybrid control problems of practical interest can be decomposed in a hierarchy of control objectives, where each objective refers to a particular time scale and to a particular level of measurement aggregation. It is common engineering practice to exploit this hierarchical structure in the development of ad hoc solutions to hybrid control problems that are far beyond the computational limitations of known methods for systematic hybrid system design. This paper extends a known design method to (i) benefit from hierarchical decompositions provided by engineering intuition, and to (ii) allow for a formal proof that the composition of the individual layers forms an overall solution.

Keywords: hierarchical control, hybrid systems, supervisory control, discrete abstraction.

1. INTRODUCTION

In the basic hybrid control configuration, continuous dynamics interact with a discrete event supervisor via a suitable interface that mediates between discrete and continuous signals. A widely accepted closed-loop model for this configuration are so called *hybrid automata* (Alur *et al.*, 2000; Henzinger, 1996). In this paper, we take the perspective of the supervisor and summarise the remaining entities as the *hybrid plant*. Control specifications are formalised as languages over the alphabet of external discrete events, and the task of the supervisor is to enforce that the closed loop evolves on acceptable trajectories according to the specification. A crucial feature of the hybrid setting is that state machine realisations of the plant typically evolve on a real-valued vector, and hence uncountable, state space. The core idea of abstraction-based approaches is that, rather than synthesising a supervisor for the actual plant behaviour, one works instead with a plant abstraction that can be realised by a finite automaton (Koutsoukos *et al.*, 2000; Cury *et al.*, 1998; Lunze *et al.*, 1997; Philips *et al.*, 1999). In (Moor and Raisch, 1999; Moor *et al.*, 2002), we develop an abstraction-based synthesis procedure within Willems' behavioural systems theory. Here, the hybrid plant is represented by its *external behaviour*, defined as the set $\mathfrak{B}_{\mathrm{p}}$ of all sequences of pairs of input and output events that are compatible with the hybrid plant dynamics. Our key result is a synthesis procedure that solves the original control problem via a *plant abstraction* $\mathfrak{B}_{\mathrm{ca}}$ with $\mathfrak{B}_{\mathrm{ca}} \supseteq \mathfrak{B}_{\mathrm{p}}$.

Although the considered abstractions $\mathfrak{B}_{\mathrm{ca}}$ are internally based on state aggregation, they are defined on the same signal space and refer to the same time scale as the original plant. On the other hand, many applications suggest an obvious decomposition of the overall control problem in a number of subproblems that refer to a hierarchy of time scales and measurement

[1] Research partially supported by the Australian Research Council, Project ID: DP0208553.

aggregations. It is common engineering practice to use this type of decomposition to find ad hoc solutions that are far beyond the computational limitations of known systematic design methods. In this paper, we extend our previous work by additional layers between plant and supervisor to represent a hierarchy of time scales, measurement aggregations, subproblems and their solutions. We develop a method that is grounded in the engineering intuition used for the hierarchical ad hoc design of hybrid systems, and additionally provide a formal proof that the composition of all individual layers solves the original problem.

The framework here is inspired by that of hierarchical DES theory (Wong and Wonham, 1996), but is technically quite distinct because we need to use an input/output structure that adequately represents both time and event driven dynamics for hybrid systems. As in other hierarchical approaches to control (Pappas *et al.*, 2000; Caines and Wei, 1998), we are concerned with the preservation of fundamental properties across levels of abstraction.

The paper is organised as follows. Section 2 summarises key results from (Moor and Raisch, 1999; Moor *et al.*, 2002). In Section 3, we present a two-level design that is readily shown to enforce the specification. The question whether the composed overall system satisfies standard admissibility conditions is more subtle. We develop sufficient criteria in Sections 4 and 5 for quasi-continuous low-level control and measurement aggregation, respectively. Section 6 extends the results to a multi-level configuration.

2. ABSTRACTION-BASED SUPERVISORY CONTROL

The purpose of this section is to briefly summarise key results of our earlier work (Moor and Raisch, 1999; Moor *et al.*, 2002) in abstraction-based supervisory controller synthesis for hybrid systems within Willems' behavioural systems theory (Willems, 1991).

Willems defines the behaviour of a dynamical system as the set of all trajectories on which the system can possibly evolve. In this paper, we restrict our considerations to the discrete-times axis $\mathbb{N}_0$:

Definition 1. A *behaviour* $\mathfrak{B}$ *over a signal space* W is a set of maps $w\colon \mathbb{N}_0 \to W$; i.e. $\mathfrak{B} \subseteq W^{\mathbb{N}_0}$. [2]

The external plant behaviour $\mathfrak{B}_\mathrm{p} \subseteq W^{\mathbb{N}_0}$ is defined as the set of all event sequences on which the hybrid plant can evolve in open loop. In (Moor *et al.*, 2001), we carefully derive $\mathfrak{B}_\mathrm{p}$ for a broad class of hybrid systems based on the *hybrid automata* model (Alur *et al.*, 2000; Henzinger, 1996) and observe that $\mathfrak{B}_\mathrm{p}$ inherits the input/output structure from the underlying continuous dynamics; i.e. we have $W := U \times Y$ and $\mathfrak{B}_\mathrm{p}$ conforms to a slightly weakened version of Willems' *I/O behaviours*:

Definition 2. A behaviour $\mathfrak{B} \subseteq W^{\mathbb{N}_0}$ to said to be a *(strict) I/- behaviour w.r.t.* (U, Y), if [3]

(i) the *input is free*, i.e. $\mathcal{P}_\mathrm{U}\mathfrak{B} = U^{\mathbb{N}_0}$ and

(ii) the *output does (strictly) not anticipate the input*, i.e.
$\mathcal{P}_\mathrm{U}\tilde{w}|_{[0,k]} = \mathcal{P}_\mathrm{U}\hat{w}|_{[0,k]} \;\Rightarrow\; (\exists\, w \in \mathfrak{B})[\, \mathcal{P}_\mathrm{Y}w|_{[0,k]} = \mathcal{P}_\mathrm{Y}\tilde{w}|_{[0,k]}$ and $\mathcal{P}_\mathrm{U}w = \mathcal{P}_\mathrm{U}\hat{w}]$
for all $k \in \mathbb{N}_0$, $\tilde{w}$, $\hat{w} \in \mathfrak{B}$; for the *strict* case the premiss on the l.h.s. is weakened to $\mathcal{P}_\mathrm{U}\tilde{w}|_{[0,k)} = \mathcal{P}_\mathrm{U}\hat{w}|_{[0,k)}$.

Adapting the concepts of supervisory control theory for DESs (Ramadge and Wonham, 1989) to the behavioural framework, the task of a supervisor $\mathfrak{B}_\mathrm{sup} \subseteq W^{\mathbb{N}_0}$ is to restrict a plant behaviour $\mathfrak{B}_\mathrm{p} \subseteq W^{\mathbb{N}_0}$ so that the closed loop is guaranteed to evolve only on acceptable signals $\mathfrak{B}_\mathrm{spec} \subseteq W^{\mathbb{N}_0}$. The closed-loop behaviour is defined by $\mathfrak{B}_\mathrm{cl} := \mathfrak{B}_\mathrm{p} \cap \mathfrak{B}_\mathrm{sup}$ and $\mathfrak{B}_\mathrm{sup}$ is said to *enforce the specification* if $\mathfrak{B}_\mathrm{cl} \subseteq \mathfrak{B}_\mathrm{spec}$.

In examining the notion of I/- behaviours, we identify two *admissibility criteria* for the interconnection of plant and supervisor: (i) any restrictions on the plant output shall only be imposed indirectly by restricting the plant input; and (ii) at any time there must be possible future evolution. Formally, we state:

Definition 3. A supervisor $\mathfrak{B}_\mathrm{sup} \subseteq W^{\mathbb{N}_0}$ is *admissible* to the plant $\mathfrak{B}_\mathrm{p} \subseteq W^{\mathbb{N}_0}$ if

(i) $\mathfrak{B}_\mathrm{sup}$ is *generically implementable*, i.e. $k \in \mathbb{N}_0$, $w|_{[0,k]} \in \mathfrak{B}_\mathrm{sup}|_{[0,k]}$, $\tilde{w}|_{[0,k]} \in W^{k+1}$, $\tilde{w}|_{[0,k]} \approx_\mathrm{y} w|_{[0,k]}$ implies $\tilde{w}|_{[0,k]} \in \mathfrak{B}_\mathrm{sup}|_{[0,k]}$; and

(ii) $\mathfrak{B}_\mathrm{p}$ and $\mathfrak{B}_\mathrm{sup}$ are *non-conflicting*, i.e. $\mathfrak{B}_\mathrm{p}|_{[0,k]} \cap \mathfrak{B}_\mathrm{sup}|_{[0,k]} = (\mathfrak{B}_\mathrm{p} \cap \mathfrak{B}_\mathrm{sup})|_{[0,k]}$ for all $k \in \mathbb{N}_0$.

This leads to the following formulation of the problem of supervisory control.

Definition 4. Given a plant $\mathfrak{B}_\mathrm{p} \subseteq W^{\mathbb{N}_0}$, $W = U \times Y$, and a specification $\mathfrak{B}_\mathrm{spec} \subseteq W^{\mathbb{N}_0}$, the pair $(\mathfrak{B}_\mathrm{p}, \mathfrak{B}_\mathrm{spec})_\mathrm{cp}$ is a *supervisory control problem*. A supervisor $\mathfrak{B}_\mathrm{sup} \subseteq W^{\mathbb{N}_0}$ that is admissible to $\mathfrak{B}_\mathrm{p}$ and that enforces $\mathfrak{B}_\mathrm{spec}$ is said to be a *solution* of $(\mathfrak{B}_\mathrm{p}, \mathfrak{B}_\mathrm{spec})_\mathrm{cp}$.

---

[2] $\mathbb{N}$ denotes the positive integers and $\mathbb{N}_0 := \mathbb{N} \cup \{0\}$. The set of all sequences in W is denoted $W^{\mathbb{N}_0} := \{w\colon \mathbb{N}_0 \to W\}$.

[3] The *restriction* operator $(\cdot)|_{[k_1,k_2)}$ maps sequences $w \in W^{\mathbb{N}_0}$ to finite strings $w|_{[k_1,k_2)} := w(k_1)w(k_1+1)\cdots w(k_2-1) \in W^{k_2-k_1}$, where we use $W^0 := \{\epsilon\}$ and ϵ denotes the *empty string*. For closed intervals, the operator $(\cdot)|_{[k_1,k_2]}$ is defined accordingly. For $W = U \times Y$, we denote $\mathcal{P}_\mathrm{U}$ and $\mathcal{P}_\mathrm{Y}$ the *natural projection* operators to the respective component, i.e. $\mathcal{P}_\mathrm{U}w = u$ and $\mathcal{P}_\mathrm{Y}w = y$ for $w = (u, y)$, $u \in U^{\mathbb{N}_0}$, $y \in Y^{\mathbb{N}_0}$. We use $\tilde{w}|_{[0,k]} \approx_\mathrm{y} w|_{[0,k]}$ as an abbreviation for the two strings to be identical up to the last output event, i.e. $\mathcal{P}_\mathrm{U}\tilde{w}|_{[0,k]} = \mathcal{P}_\mathrm{U}w|_{[0,k]}$ and $\mathcal{P}_\mathrm{Y}\tilde{w}|_{[0,k)} = \mathcal{P}_\mathrm{Y}w|_{[0,k)}$.

If both $\mathfrak{B}_{\mathrm{p}}$ and $\mathfrak{B}_{\mathrm{spec}}$ were realised by finite automata, the *least restrictive solution* of $(\mathfrak{B}_{\mathrm{p}},\ \mathfrak{B}_{\mathrm{spec}})_{\mathrm{cp}}$ could be readily computed. While a finite automaton realisation for $\mathfrak{B}_{\mathrm{spec}}$ is a modest requirement, the hybrid plant in general is *not* realisable on a finite state space. We approach the problem by replacing $\mathfrak{B}_{\mathrm{p}}$ with an *abstraction* $\mathfrak{B}_{\mathrm{ca}}$ (so $\mathfrak{B}_{\mathrm{p}} \subseteq \mathfrak{B}_{\mathrm{ca}}$) that is realised by a finite automaton; so we can establish a solution $\mathfrak{B}_{\mathrm{sup}}$ of $(\mathfrak{B}_{\mathrm{ca}},\ \mathfrak{B}_{\mathrm{spec}})_{\mathrm{cp}}$. Clearly, $\mathfrak{B}_{\mathrm{sup}}$ enforces the specification for the original plant: $\mathfrak{B}_{\mathrm{p}} \cap \mathfrak{B}_{\mathrm{sup}} \subseteq \mathfrak{B}_{\mathrm{p}} \cap \mathfrak{B}_{\mathrm{ca}} \subseteq \mathfrak{B}_{\mathrm{spec}}$. An argument that shows that $\mathfrak{B}_{\mathrm{sup}}$ also is admissible to $\mathfrak{B}_{\mathrm{p}}$ can be based on the following notion of *completeness*:

Definition 5. A behaviour $\mathfrak{B} \subseteq W^{\mathbb{N}_0}$ is *complete* if
$$w \in \mathfrak{B} \quad \Leftrightarrow \quad \forall\, k \in \mathbb{N}_0 :\ w|_{[0,k]} \in \mathfrak{B}|_{[0,k]}\,.$$

By the following proposition, admissibility of a supervisor is independent of the particular plant dynamics provided that all involved behaviours are complete.

Proposition 6. Let $\mathfrak{B}_{\mathrm{p}} \subseteq W^{\mathbb{N}_0}$ be a complete I/- behaviour and $\mathfrak{B}_{\mathrm{sup}} \subseteq W^{\mathbb{N}_0}$ be complete and generically implementable. Then $\mathfrak{B}_{\mathrm{p}}$ and $\mathfrak{B}_{\mathrm{sup}}$ are nonconflicting.

For the rest of this paper, we restrict consideration to complete behaviours, and we obtain our main result for abstraction-based supervisory control as a consequence of Proposition 6.

Theorem 7. Let $\mathfrak{B}_{\mathrm{ca}} \subseteq W^{\mathbb{N}_0}$, $W = U \times Y$, be an abstraction of an I/- behaviour $\mathfrak{B}_{\mathrm{p}} \subseteq W^{\mathbb{N}_0}$, let $\mathfrak{B}_{\mathrm{spec}} \subseteq W^{\mathbb{N}_0}$, and let $\mathfrak{B}_{\mathrm{sup}} \subseteq W^{\mathbb{N}_0}$ be a solution to the supervisory control problem $(\mathfrak{B}_{\mathrm{ca}},\ \mathfrak{B}_{\mathrm{spec}})_{\mathrm{cp}}$. If $\mathfrak{B}_{\mathrm{p}}$ and $\mathfrak{B}_{\mathrm{sup}}$ are complete then $\mathfrak{B}_{\mathrm{sup}}$ is a solution of $(\mathfrak{B}_{\mathrm{p}},\ \mathfrak{B}_{\mathrm{spec}})_{\mathrm{cp}}$.

3. A TWO-LEVEL BOTTOM-UP DESIGN

We motivate our approach with a mobile robot scenario, in which a robot shall patrol some area. Suppose we are given the robot behaviour $\mathfrak{B}^{\mathrm{L}}_{\mathrm{p}}$ over a signal space $W_{\mathrm{L}} = U_{\mathrm{L}} \times Y_{\mathrm{L}}$, where U_{L} represents low-level inputs for acceleration and Y_{L} represents velocity and position. The control objective can be represented by the set $\mathfrak{B}^{\mathrm{L}}_{\mathrm{spec}} \subseteq W_{\mathrm{L}}^{\mathbb{N}_0}$ of all signals that correspond to some motion that we regard as an acceptable patroling behaviour, e.g. we may partition the area and consider all paths as acceptable that pass through the partition blocks in a cyclic fashion. This leads to the control problem $(\mathfrak{B}^{\mathrm{L}}_{\mathrm{p}},\ \mathfrak{B}^{\mathrm{L}}_{\mathrm{spec}})_{\mathrm{cp}}$ and we seek a solution $\mathfrak{B}^{\mathrm{L}}_{\mathrm{sup}} \subseteq W_{\mathrm{L}}^{\mathbb{N}_0}$. However, a reasonably accurate plant model $\mathfrak{B}^{\mathrm{L}}_{\mathrm{p}}$ will be based on the mechanics of the robot, and it appears impractical to solve $(\mathfrak{B}^{\mathrm{L}}_{\mathrm{p}},\ \mathfrak{B}^{\mathrm{L}}_{\mathrm{spec}})_{\mathrm{cp}}$ "in one go". Engineering intuition suggests that we first design a family of low-level controllers that implement elementary manoeuvres like "move forward", "turn right", "turn left". In a second design step, we ask for a high-level supervisor to schedule the elementary manoeuvres to achieve the desired patroling behaviour. The high-level supervisor $\mathfrak{B}^{\mathrm{H}}_{\mathrm{sup}} \subseteq W_{\mathrm{H}}^{\mathbb{N}_0}$ operates on a high-level signal space $W_{\mathrm{H}} = U_{\mathrm{H}} \times Y_{\mathrm{H}}$, where each control action in U_{H} selects a particular low-level controller and the measurement events in Y_{H} correspond to a coarse quantisation of the robot position, perhaps based on the partition blocks used for the statement of $\mathfrak{B}^{\mathrm{L}}_{\mathrm{spec}}$.

The relationship between low-level and high-level signals is formally represented by a behaviour $\mathfrak{B}_{\mathrm{im}}$ over $W_{\mathrm{H}} \times W_{\mathrm{L}} = U_{\mathrm{H}} \times Y_{\mathrm{H}} \times U_{\mathrm{L}} \times Y_{\mathrm{L}}$; see Figure 1. Note that the behaviour $\mathfrak{B}_{\mathrm{im}}$ may itself exhibit nontrivial dynamics and therefore provides a universal tool to link the two levels.

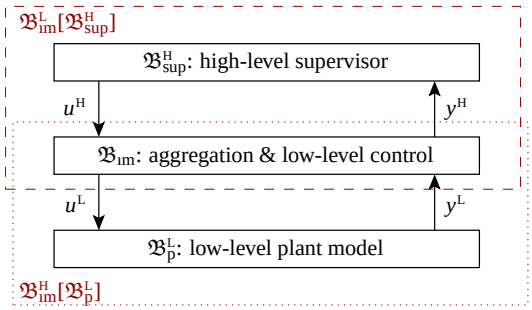

Fig. 1. Plant (supervisor) perspective, dashed (dotted)

From the perspective of the low-level plant $\mathfrak{B}^{\mathrm{L}}_{\mathrm{p}}$, the interconnection of $\mathfrak{B}_{\mathrm{im}}$ with $\mathfrak{B}^{\mathrm{H}}_{\mathrm{sup}}$ plays the role of a compound low-level supervisor $\mathfrak{B}^{\mathrm{L}}_{\mathrm{im}}[\mathfrak{B}^{\mathrm{H}}_{\mathrm{sup}}]$ over W_{L}, as indicated by the dashed box in Figure 1. The external behaviour $\mathfrak{B}^{\mathrm{L}}_{\mathrm{im}}[\mathfrak{B}^{\mathrm{H}}_{\mathrm{sup}}]$ is given by the projection of $\mathfrak{B}_{\mathrm{im}}$ into $W_{\mathrm{L}}^{\mathbb{N}_0}$ with the internal high-level signal restricted to $\mathfrak{B}^{\mathrm{H}}_{\mathrm{sup}}$:

$$\mathfrak{B}^{\mathrm{L}}_{\mathrm{im}}[\mathfrak{B}^{\mathrm{H}}_{\mathrm{sup}}] := \{w^{\mathrm{L}} |\ (\exists\, w^{\mathrm{H}} \in \mathfrak{B}^{\mathrm{H}}_{\mathrm{sup}})[\ (w^{\mathrm{H}},\ w^{\mathrm{L}}) \in \mathfrak{B}_{\mathrm{im}}\ ]\}. \quad (1)$$

In addressing the original problem $(\mathfrak{B}^{\mathrm{L}}_{\mathrm{p}},\ \mathfrak{B}^{\mathrm{L}}_{\mathrm{spec}})_{\mathrm{cp}}$, we require that $\mathfrak{B}^{\mathrm{L}}_{\mathrm{im}}[\mathfrak{B}^{\mathrm{H}}_{\mathrm{sup}}]$ solves $(\mathfrak{B}^{\mathrm{L}}_{\mathrm{p}},\ \mathfrak{B}^{\mathrm{L}}_{\mathrm{spec}})_{\mathrm{cp}}$, and, in particular, enforces the specification:

$$\mathfrak{B}^{\mathrm{L}}_{\mathrm{p}} \cap \mathfrak{B}^{\mathrm{L}}_{\mathrm{im}}[\mathfrak{B}^{\mathrm{H}}_{\mathrm{sup}}] \subseteq \mathfrak{B}^{\mathrm{L}}_{\mathrm{spec}}\,. \quad (2)$$

From the perspective of the high-level supervisor $\mathfrak{B}^{\mathrm{H}}_{\mathrm{sup}}$, we obtain from interconnecting $\mathfrak{B}_{\mathrm{im}}$ with $\mathfrak{B}^{\mathrm{L}}_{\mathrm{p}}$ a compound high-level plant $\mathfrak{B}^{\mathrm{H}}_{\mathrm{im}}[\mathfrak{B}^{\mathrm{L}}_{\mathrm{p}}]$ over W_{H} (dotted box in Figure 1):

$$\mathfrak{B}^{\mathrm{H}}_{\mathrm{im}}[\mathfrak{B}^{\mathrm{L}}_{\mathrm{p}}] := \{w^{\mathrm{H}} |\ (\exists\, w^{\mathrm{L}} \in \mathfrak{B}^{\mathrm{L}}_{\mathrm{p}})[\ (w^{\mathrm{H}},\ w^{\mathrm{L}}) \in \mathfrak{B}_{\mathrm{im}}\ ]\}\,. \quad (3)$$

Although we are not given a high-level specification, we still need to fulfil the admissibility criteria for system interconnection: $\mathfrak{B}^{\mathrm{H}}_{\mathrm{sup}}$ must be generically implementable, and $\mathfrak{B}^{\mathrm{H}}_{\mathrm{im}}[\mathfrak{B}^{\mathrm{L}}_{\mathrm{p}}]$ and $\mathfrak{B}^{\mathrm{H}}_{\mathrm{sup}}$ must be nonconflicting. We summarise our discussion of Figure 1:

Definition 8. The pair $(\mathfrak{B}_{\mathrm{im}},\ \mathfrak{B}^{\mathrm{H}}_{\mathrm{sup}})_{\mathrm{tl}}$ is a *two-level hierarchical solution* to the supervisory control problem $(\mathfrak{B}^{\mathrm{L}}_{\mathrm{p}},\ \mathfrak{B}^{\mathrm{L}}_{\mathrm{spec}})_{\mathrm{cp}}$ if

(i) $\mathfrak{B}^{\mathrm{L}}_{\mathrm{p}} \cap \mathfrak{B}^{\mathrm{L}}_{\mathrm{im}}[\mathfrak{B}^{\mathrm{H}}_{\mathrm{sup}}] \subseteq \mathfrak{B}^{\mathrm{L}}_{\mathrm{spec}}$, and
(iia) $\mathfrak{B}^{\mathrm{L}}_{\mathrm{im}}[\mathfrak{B}^{\mathrm{H}}_{\mathrm{sup}}]$ is admissible to $\mathfrak{B}^{\mathrm{L}}_{\mathrm{p}}$, and
(iib) $\mathfrak{B}^{\mathrm{H}}_{\mathrm{sup}}$ is admissible to $\mathfrak{B}^{\mathrm{H}}_{\mathrm{im}}[\mathfrak{B}^{\mathrm{L}}_{\mathrm{p}}]$.

We are now in the position to recover the intuitive bottom-up-design motivated by the mobile robot scenario. In a first step, we represent the *intended* relationship between high-level signals and low-level signals. Formally, let $\mathfrak{B}^{\mathrm{HL}}_{\mathrm{spec}} \subseteq (W_{\mathrm{H}} \times W_{\mathrm{L}})^{\mathbb{N}_0}$ denote the set of all signal pairs $(w^{\mathrm{H}},\ w^{\mathrm{L}})$ that conform with the desired effect of high-level control actions on the low-level plant $\mathfrak{B}^{\mathrm{L}}_{\mathrm{p}}$, and the desired scheme of measurement aggregation to generate high-level measurement events from low-level signals. We then ask the intermediate layer $\mathfrak{B}_{\mathrm{im}}$ to enforce the specification $\mathfrak{B}^{\mathrm{HL}}_{\mathrm{spec}}$ when interconnected with the low-level plant $\mathfrak{B}^{\mathrm{L}}_{\mathrm{p}}$. This condition is expressed by the following inclusion:

$$\{(w^{\mathrm{H}},\ w^{\mathrm{L}}) \in \mathfrak{B}_{\mathrm{im}} |\ w^{\mathrm{L}} \in \mathfrak{B}^{\mathrm{L}}_{\mathrm{p}}\} \subseteq \mathfrak{B}^{\mathrm{HL}}_{\mathrm{spec}}\,. \qquad (4)$$

Suppose we have designed $\mathfrak{B}_{\mathrm{im}}$ according to Eq. (4) and, in a second step, we want to design $\mathfrak{B}^{\mathrm{H}}_{\mathrm{sup}}$. Thus, we are looking for an abstraction $\tilde{\mathfrak{B}}^{\mathrm{H}}_{\mathrm{p}}$ of the high-level plant $\mathfrak{B}^{\mathrm{H}}_{\mathrm{im}}[\mathfrak{B}^{\mathrm{L}}_{\mathrm{p}}]$ and for a high-level specification $\tilde{\mathfrak{B}}^{\mathrm{H}}_{\mathrm{spec}}$ that expresses $\mathfrak{B}^{\mathrm{L}}_{\mathrm{spec}}$ in terms of high-level signals. Both can be obtained from Eq. (4):

$$\tilde{\mathfrak{B}}^{\mathrm{H}}_{\mathrm{p}} := \{w^{\mathrm{H}} |\ (\exists\, w^{\mathrm{L}})[\ (w^{\mathrm{H}},\ w^{\mathrm{L}}) \in \mathfrak{B}^{\mathrm{HL}}_{\mathrm{spec}}]\}\,; \qquad (5)$$

$$\tilde{\mathfrak{B}}^{\mathrm{H}}_{\mathrm{spec}} := \{w^{\mathrm{H}} |\ (\forall\, w^{\mathrm{L}})[\ (w^{\mathrm{H}},\ w^{\mathrm{L}}) \in \mathfrak{B}^{\mathrm{HL}}_{\mathrm{spec}} \ \Rightarrow\ w^{\mathrm{L}} \in \mathfrak{B}^{\mathrm{L}}_{\mathrm{spec}}\ ]\}\,. \qquad (6)$$

Observe that the control problem $(\tilde{\mathfrak{B}}^{\mathrm{H}}_{\mathrm{p}},\ \tilde{\mathfrak{B}}^{\mathrm{H}}_{\mathrm{spec}})_{\mathrm{cp}}$ does *not* depend on the actual low-level plant under low-level control $\mathfrak{B}^{\mathrm{H}}_{\mathrm{im}}[\mathfrak{B}^{\mathrm{L}}_{\mathrm{p}}]$, but only on the intended outcome $\mathfrak{B}^{\mathrm{HL}}_{\mathrm{spec}}$ of the preceeding low-level design. In our motivational robot scenario, $\mathfrak{B}^{\mathrm{HL}}_{\mathrm{spec}}$ may be modelled by a *linear hybrid automata* (Alur *et al.*, 2000) in which a two dimensional polyhedral differential inclusion specifies constraints on the continuous evolution of the robot's position, *reset relations* abstract motion during the settling time of the low-level controllers, and *mode invariants* correspond to the measurement aggregation in that the high-level supervisor is only notified of discrete *mode transitions*. A high-level supervisor $\mathfrak{B}^{\mathrm{H}}_{\mathrm{sup}}$ that solves $(\tilde{\mathfrak{B}}^{\mathrm{H}}_{\mathrm{p}},\ \tilde{\mathfrak{B}}^{\mathrm{H}}_{\mathrm{spec}})_{\mathrm{cp}}$ can then be computed efficiently, e.g. using the methods presented in (Moor and Raisch, 1999; Moor *et al.*, 2002).

In general, the choice of $\mathfrak{B}^{\mathrm{HL}}_{\mathrm{spec}}$ can be guided by the same engineering intuition that we would use in a hierarchical ad hoc design. The contribution here is to develop a framework in which we can formally prove that the composition of a high-level controller with an intermediate layer forms a solution of the original problem. Proposition 9 provides a first step in this proof and shows that a high-level supervisor design based on $(\tilde{\mathfrak{B}}^{\mathrm{H}}_{\mathrm{p}},\ \tilde{\mathfrak{B}}^{\mathrm{H}}_{\mathrm{spec}})_{\mathrm{cp}}$ satisfies requirement (i) in Definition 8: the overall configuration enforces the low-level specification. In the following sections, we address the admissibility criteria (iia) and (iib).

Proposition 9. Any solution $\mathfrak{B}^{\mathrm{H}}_{\mathrm{sup}}$ of $(\tilde{\mathfrak{B}}^{\mathrm{H}}_{\mathrm{p}},\ \tilde{\mathfrak{B}}^{\mathrm{H}}_{\mathrm{spec}})_{\mathrm{cp}}$ satisfies $\mathfrak{B}^{\mathrm{L}}_{\mathrm{p}} \cap \mathfrak{B}^{\mathrm{L}}_{\mathrm{im}}[\mathfrak{B}^{\mathrm{H}}_{\mathrm{sup}}] \subseteq \mathfrak{B}^{\mathrm{L}}_{\mathrm{spec}}$.

4. ADMISSIBILITY: UNIFORM TIME SCALES

What properties should we ask for $\mathfrak{B}_{\mathrm{im}}$ in order to satisfy the admissibility criteria in Definition 8? In this section, we examine the case of *quasi-continuous* controllers, i.e. controllers that have been designed by continuous methods but are technically realised by digital hardware at a reasonably high sampling rate and a comparatively fine quantisation. An important feature of this setting is a uniform time scale on all signals, e.g. $y^{\mathrm{L}}(k)$ takes place at the same physical time as $y^{\mathrm{H}}(k)$.

Natural candidates for $\mathfrak{B}_{\mathrm{im}}$ are I/- behaviours, where u^{H} and y^{L} play the role of the input to $\mathfrak{B}_{\mathrm{im}}$ while y^{H} and u^{L} are considered outputs. In addition, we require that $\mathfrak{B}_{\mathrm{im}}$ and $\mathfrak{B}^{\mathrm{L}}_{\mathrm{p}}$ be complete, and from this conclude that the I/- property of $\mathfrak{B}^{\mathrm{L}}_{\mathrm{p}}$ is passed on to $\mathfrak{B}^{\mathrm{H}}_{\mathrm{im}}[\mathfrak{B}^{\mathrm{L}}_{\mathrm{p}}]$. In order to derive completeness of $\mathfrak{B}^{\mathrm{H}}_{\mathrm{im}}[\mathfrak{B}^{\mathrm{L}}_{\mathrm{p}}]$, we require that all signal spaces are finite sets.

Lemma 10. If $\mathfrak{B}_{\mathrm{im}}$ is a complete strict I/- behaviour w.r.t. $(U_{\mathrm{H}} \times Y_{\mathrm{L}},\ Y_{\mathrm{H}} \times U_{\mathrm{L}})$, and if $\mathfrak{B}^{\mathrm{L}}_{\mathrm{p}}$ is a complete I/- behaviour w.r.t. $(U_{\mathrm{L}},\ Y_{\mathrm{L}})$, then $\mathfrak{B}^{\mathrm{H}}_{\mathrm{im}}[\mathfrak{B}^{\mathrm{L}}_{\mathrm{p}}]$ is a complete I/- behaviour w.r.t. $(U_{\mathrm{H}},\ Y_{\mathrm{H}})$.

The same criteria not only preserves the I/- structure of $\mathfrak{B}^{\mathrm{L}}_{\mathrm{p}}$ but also generic implementability of $\mathfrak{B}^{\mathrm{H}}_{\mathrm{sup}}$.

Lemma 11. If $\mathfrak{B}_{\mathrm{im}}$ is a complete strict I/- behaviour w.r.t. $(U_{\mathrm{H}} \times Y_{\mathrm{L}},\ Y_{\mathrm{H}} \times U_{\mathrm{L}})$, and if $\mathfrak{B}^{\mathrm{H}}_{\mathrm{sup}}$ is complete and generically implementable, then $\mathfrak{B}^{\mathrm{L}}_{\mathrm{im}}[\mathfrak{B}^{\mathrm{H}}_{\mathrm{sup}}]$ is complete and generically implementable.

Suppose we have found a complete solution $\mathfrak{B}^{\mathrm{H}}_{\mathrm{sup}}$ of $(\tilde{\mathfrak{B}}^{\mathrm{H}}_{\mathrm{p}},\ \tilde{\mathfrak{B}}^{\mathrm{H}}_{\mathrm{spec}})_{\mathrm{cp}}$, and suppose that $\mathfrak{B}^{\mathrm{HL}}_{\mathrm{spec}}$ is implemented by a complete strict I/- behaviour $\mathfrak{B}_{\mathrm{im}}$. Under the hypothesis of Lemmata 11 and 10 we conclude together with Proposition 6 that the admissibility criteria (iia) and (iib), Definition 8, are satisfied. Hence, by Proposition 9, the pair $(\mathfrak{B}_{\mathrm{im}},\ \mathfrak{B}^{\mathrm{H}}_{\mathrm{sup}})_{\mathrm{tl}}$ is a two-level hierarchical solution of $(\mathfrak{B}^{\mathrm{L}}_{\mathrm{p}},\ \mathfrak{B}^{\mathrm{L}}_{\mathrm{spec}})_{\mathrm{cp}}$.

5. ADMISSIBILITY: DIFFERENT TIME SCALES

The proposed high-level measurement signal in the robot scenario shall notify the high-level supervisor whenever the robot position passes into a different partition cell. Therefore, high-level and low-level signals refer to different time scales. Furthermore, the relationship between the high-level and low-level timing is *not* determined by a fixed factor but is event driven by the low-level measurement signal. We develop an

internal structure for $\mathfrak{B}_{\mathrm{im}}$ that implements this dynamic relationship as a general tool of measurement aggregation.

The following definition extends the basic notion of causal maps (e.g. (Khalil, 1996)) in referring to different time scales for cause and effect, respectively:

Definition 12. Let $F: U^{\mathbb{N}_0} \to Y^{\mathbb{N}_0}$ and $T: U^{\mathbb{N}_0} \to \mathbb{N}_0{}^{\mathbb{N}_0}$. The operator F is said to be *causal* if

$$\tilde{u}|_{[0,k]} = \hat{u}|_{[0,k]} \quad \Rightarrow \quad F(\tilde{u})|_{[0,k]} = F(\hat{u})|_{[0,k]} \quad (7)$$

for all $k \in \mathbb{N}_0$, $\tilde{u}, \hat{u} \in U^{\mathbb{N}_0}$. The operator F is said to be *strictly causal* if

$$\tilde{u}|_{[0,k)} = \hat{u}|_{[0,k)} \quad \Rightarrow \quad F(\tilde{u})|_{[0,k]} = F(\hat{u})|_{[0,k]} \quad (8)$$

for all $k \in \mathbb{N}_0$, $\tilde{u}, \hat{u} \in U^{\mathbb{N}_0}$. The operator T is said to be a *dynamic time scale* if T is strictly causal and if the *time transformation* $T(u): \mathbb{N}_0 \to \mathbb{N}_0$ is surjective and monotone increasing for all $u \in U^{\mathbb{N}_0}$. The operator F is said to be *causal w.r.t.* T if T is a dynamic time scale and if

$$\tilde{u}|_{[0,j]} = \hat{u}|_{[0,j]} \quad \Rightarrow \quad F(\tilde{u})|_{[0,k]} = F(\hat{u})|_{[0,k]} \quad (9)$$

for $k = T(\tilde{u})(j)$ and all $j \in \mathbb{N}_0$, $\tilde{u}, \hat{u} \in U^{\mathbb{N}_0}$.

For a fixed input u, the time transformation $T(u)$ maps low-level time $j \in \mathbb{N}_0$ to high-level time $k \in \mathbb{N}_0$. By requiring that T itself is a strictly causal operator, we ensure that at any instant of time the transformation $T(u)$ only depends on the strict past of u.

In our target application, the low-level measurement signal y^{L} plays the role of the input u and drives the time transformation $T(y^{\mathrm{L}})$. The high-level measurement is generated by $y^{\mathrm{H}} = F(y^{\mathrm{L}})$ where the operator $F: Y_{\mathrm{L}}^{\mathbb{N}_0} \to Y_{\mathrm{H}}^{\mathbb{N}_0}$ is required to be causal w.r.t. T. As an example for a realisation of F, consider an automaton that generates high-level events whenever the low-level measurement equals a given value or completes a given cycle.

We relate high-level controls u^{H} and low-level controls u^{L} by a sample-and-hold device that is triggered by the time transformation $T(y^{\mathrm{L}})$; i.e. successive high-level control actions are passed on to the lower level whenever a high-level measurement is generated. Formally, this is expressed by $u^{\mathrm{L}} = u^{\mathrm{H}} \circ T(y^{\mathrm{L}})$.

In summary, our candidate $\mathfrak{B}_{\mathrm{im}}$ is constructed from a dynamic time scale T and an operator F that is causal w.r.t. T. Figure 2 illustrates the formal definition:

$$\begin{aligned}\mathfrak{B}_{\mathrm{im}} := \{(u^{\mathrm{H}}, y^{\mathrm{H}}, u^{\mathrm{L}}, y^{\mathrm{L}}) | \\ y^{\mathrm{H}} = F(y^{\mathrm{L}}) \text{ and } u^{\mathrm{L}} = u^{\mathrm{H}} \circ T(y^{\mathrm{L}})\}. \end{aligned} \quad (10)$$

Our candidate $\mathfrak{B}_{\mathrm{im}}$ turns out to be complete:

Proposition 13. Given a dynamic time scale $T: Y_{\mathrm{L}}^{\mathbb{N}_0} \to \mathbb{N}_0{}^{\mathbb{N}_0}$ and an operator $F: Y_{\mathrm{L}}^{\mathbb{N}_0} \to Y_{\mathrm{H}}^{\mathbb{N}_0}$ that is causal w.r.t. T, define $\mathfrak{B}_{\mathrm{im}}$ by Eq. (10). Then $\mathfrak{B}_{\mathrm{im}}$ is complete.

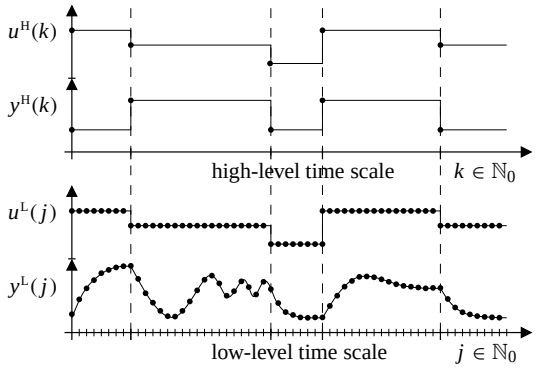

Fig. 2. Relation between time scales in $\mathfrak{B}_{\mathrm{im}}$

Analogous to the results in Section 4, the candidate $\mathfrak{B}_{\mathrm{im}}$ from Eq. (10) preserves the input/output structure of a plant and generic implementability of a supervisor.

Lemma 14. Under the hypothesis of Proposition 13, and if $\mathfrak{B}_{\mathrm{p}}^{\mathrm{L}}$ is a complete I/- behaviour w.r.t. $(U_{\mathrm{L}}, Y_{\mathrm{L}})$, it follows that $\mathfrak{B}_{\mathrm{im}}^{\mathrm{H}}[\mathfrak{B}_{\mathrm{p}}^{\mathrm{L}}]$ is a complete I/- behaviour w.r.t. $(U_{\mathrm{H}}, Y_{\mathrm{H}})$.

Lemma 15. Under the hypothesis of Proposition 13, and provided that $\mathfrak{B}_{\mathrm{sup}}^{\mathrm{H}}$ is complete and generically implementable, it follows that $\mathfrak{B}_{\mathrm{im}}^{\mathrm{L}}[\mathfrak{B}_{\mathrm{sup}}^{\mathrm{H}}]$ is complete and generically implementable.

Along the same line of thought as in the previous section, Lemmata 14 and 15 can be used to show that if the intermediate specification $\mathfrak{B}_{\mathrm{spec}}^{\mathrm{HL}}$ is implemented through a behaviour $\mathfrak{B}_{\mathrm{im}}$ according to Eq. (10), the pair $(\mathfrak{B}_{\mathrm{im}}, \mathfrak{B}_{\mathrm{sup}}^{\mathrm{H}})_{\mathrm{tl}}$ is a two-level hierarchical solution of $(\mathfrak{B}_{\mathrm{p}}^{\mathrm{L}}, \mathfrak{B}_{\mathrm{spec}}^{\mathrm{L}})_{\mathrm{cp}}$.

6. MULTI-LEVEL HIERARCHICAL DESIGN

To treat typical hybrid control configurations, we would like to combine at least two intermediate layers for low-level control and measurement aggregation, respectively. In this section, we show that our results readily extend to a multi-level configuration.

Let $W_i = U_i \times Y_i$ denote the signal space on the i-th level, $0 \le i \le m$, and consider a low-level plant $\mathfrak{B}_{\mathrm{p}}^0$ over W_0, intermediate layers $\mathfrak{B}_{\mathrm{im}}^{i,i-1}$ over $W_i \times W_{i-1}$, $1 \le i \le m$, and a high-level supervisor $\mathfrak{B}_{\mathrm{sup}}^m$ over W_m. We assume that $\mathfrak{B}_{\mathrm{p}}^0$ is a complete I/- behaviour, that $\mathfrak{B}_{\mathrm{sup}}^m$ is complete and generically implementable, and that each intermediate layer is of either type discussed in Section 4 and 5. For the levels i, $0 \le i < m$, we iteratively define the behaviour from the plant perspective

$$\mathfrak{B}_{\mathrm{sup}}^i := \{w^i | \exists\, w^{i+1} \in \mathfrak{B}_{\mathrm{sup}}^{i+1} : (w^{i+1}, w^i) \in \mathfrak{B}_{\mathrm{im}}^{i+1,i}\},$$

and, for i, $0 < i \le m$, from the supervisor perspective:

$$\mathfrak{B}_{\mathrm{p}}^i := \{w^i | \exists\, w^{i-1} \in \mathfrak{B}_{\mathrm{p}}^{i-1} : (w^i, w^{i-1}) \in \mathfrak{B}_{\mathrm{im}}^{i,i-1}\}.$$

Definition 16. The tuple $(\mathfrak{B}_{\text{im}}^{1,0}, \mathfrak{B}_{\text{im}}^{2,1}, \ldots \mathfrak{B}_{\text{im}}^{m,m-1}, \mathfrak{B}_{\text{sup}}^{m})_{\text{ml}}$ is said to be an *(m+1)-level hierarchical solution* to the control problem $(\mathfrak{B}_{\text{p}}^{0}, \mathfrak{B}_{\text{spec}}^{0})_{\text{cp}}$ if

(i) $\mathfrak{B}_{\text{p}}^{0} \cap \mathfrak{B}_{\text{sup}}^{0} \subseteq \mathfrak{B}_{\text{spec}}^{0}$, and

(ii) for all i, $0 \leq i \leq m$, $\mathfrak{B}_{\text{sup}}^{i}$ is admissible to $\mathfrak{B}_{\text{p}}^{i}$.

We propose an iterative bottom-up design. Assume that, after the design of n layers, we were given behaviours over appropriate signal spaces such that for all i, $0 < i \leq n$:

(a) $\mathfrak{B}_{\text{p}}^{i} \subseteq \tilde{\mathfrak{B}}_{\text{p}}^{i}$,

(b) $\{(w^i, w^{i-1}) \in \mathfrak{B}_{\text{im}}^{i,i-1} \mid w^{i-1} \in \tilde{\mathfrak{B}}_{\text{p}}^{i-1}\} \subseteq \mathfrak{B}_{\text{spec}}^{i,i-1}$,

(c) $w^i \in \mathfrak{B}_{\text{spec}}^{i}$ and $(w^i, w^{i-1}) \in \mathfrak{B}_{\text{spec}}^{i,i-1}$
$\Rightarrow \quad w^{i-1} \in \mathfrak{B}_{\text{spec}}^{i-1}$.

We design the $(n+1)$-th layer to satisfy a specification $\mathfrak{B}_{\text{spec}}^{n+1,n}$ over $W_{n+1} \times W_n$. Suppose we can enforce the specification for the plant abstraction $\tilde{\mathfrak{B}}_{\text{p}}^{n}$; i.e. we find $\mathfrak{B}_{\text{im}}^{n+1,n}$ that satisfies (b) at $i = n+1$. As an abstraction for $\mathfrak{B}_{\text{p}}^{n+1}$ we use $\tilde{\mathfrak{B}}_{\text{p}}^{n+1} := \mathcal{P}_{W_{n+1}} \mathfrak{B}_{\text{spec}}^{n+1,n}$, to satisfy (a) at $i = n+1$. Finally, we choose

$$\mathfrak{B}_{\text{spec}}^{n+1} := \{w^{n+1} \mid (\forall\, w^n)\, [\, (w^{n+1},\, w^n) \in \mathfrak{B}_{\text{spec}}^{n+1,n} \;\Rightarrow\; w^n \in \mathfrak{B}_{\text{spec}}^{n}\,]\}. \quad (11)$$

Clearly our choice of $\mathfrak{B}_{\text{spec}}^{n+1}$ satisfies (c). Hence, all requirements are satisfied at $i = n+1$ and we can iterate the procedure for $n = 1, 2, \ldots m$. As top-level supervisor $\mathfrak{B}_{\text{sup}}^{m}$ we use a complete solution of $(\tilde{\mathfrak{B}}_{\text{p}}^{m}, \mathfrak{B}_{\text{spec}}^{m})_{\text{cp}}$. It is readily seen that $\mathfrak{B}_{\text{p}}^{0} \cap \mathfrak{B}_{\text{sup}}^{0} \subseteq \mathfrak{B}_{\text{spec}}^{0}$ and, hence, the design satisfies condition (i).

To verify condition (ii), we invoke the lemmata from the two previous sections. By successive application of Lemmata 10 and 14 for increasing i, the completeness and the I/- property of $\mathfrak{B}_{\text{p}}^{i}$ is inherited by $\mathfrak{B}_{\text{p}}^{i+1}$. Similarly, by successive application of Lemmata 11 and 15 for decreasing i, the completeness and the generic implementability of $\mathfrak{B}_{\text{sup}}^{i}$ passes on to $\mathfrak{B}_{\text{sup}}^{i-1}$. By Proposition 6, we conclude that (ii) is satisfied.

7. CONCLUSION

In this paper, we extend the behavioural framework to hybrid system synthesis (Moor and Raisch, 1999; Moor *et al.*, 2002) with additional layers between plant and supervisor to represent a hierarchy of time scales, measurement aggregations, subproblems and their solutions. Technically, our main contribution are sufficient criteria that guarantee standard admissibility conditions for the hierarchical composition of the overall control system.

Although we do account for internal continuous dynamics, all external signals are event sequences. This facilitates the discussion and we can focus our efforts on the hierarchical architecture. We argue that in engineering realisations of complex control architectures virtually all continuous controllers will be implemented by digital hardware in a quasi-continuous setting. This is covered by our framework, which facilitates the usage of both traditional continuous techniques for time driven dynamics and DES techniques for event driven dynamics.

8. REFERENCES

Alur, R., T.A. Henzinger, G. Lafferriere and G. Pappas (2000). Discrete abstractions of hybrid systems. *Proc. of the IEEE* **88**, 971–984.

Caines, P.E. and Y.J. Wei (1998). Hierarchical hybrid control systems: a lattice theoretic formulation. *IEEE Trans. Automat. Contr.* **43:4**, 501–508.

Cury, J.E.R., B.A. Krogh and T. Niinomi (1998). Synthesis of supervisory controllers for hybrid systems based on approximating automata. *IEEE Trans. Autom. Cont.* **43**, 564–568.

Henzinger, T.A. (1996). The theory of hybrid automata. In: *11th Annual IEEE Symposium on Logic in Computer Science*. pp. 278–292.

Khalil, H.K. (1996). *Nonlinear Systems*. Prentice-Hall. Second edition.

Koutsoukos, X., P.J. Antsaklis, J.A. Stiver and M.D. Lemmon (2000). Supervisory control of hybrid systems. *Proc. of the IEEE* **88**, 1026–1049.

Lunze, J., B. Nixdorf and H. Richter (1997). Hybrid modelling of continuous-variable systems with application to supervisory control. In: *Proc. European Control Conference*.

Moor, T. and J. Raisch (1999). Supervisory control of hybrid systems within a behavioural framework. *Systems and Control Letters* **38**, 157–166.

Moor, T., J. Raisch and J.M. Davoren (2001). Computational advantages of a two-level hybrid control architecture. In: *Proc. 40th IEEE Conf. Decision and Control*. pp. 358–362.

Moor, T., J. Raisch and S.D. O'Young (2002). Discrete supervisory control of hybrid systems based on l-complete approximations. *Discrete Event Dynamic Systems* **12**, 83–107.

Pappas, G.J., G. Lafferriere and S. Sastry (2000). Hierarchically consistent control systems. *IEEE Trans. Autom. Contr.* **45:6**, 1144–1160.

Philips, P., M. Weiss and H.A. Preisig (1999). Control based on discrete-event models of continuous systems. In: *Proc. European Control Conference*. Karlsruhe, Germany.

Ramadge, P.J. and W.M. Wonham (1989). The control of discrete event systems. *Proc. of the IEEE* **77**, 81–98.

Willems, J.C. (1991). Paradigms and puzzles in the theory of dynamic systems. *IEEE Trans. Autom. Contr.* **36**, 258–294.

Wong, K.C. and W.M. Wonham (1996). Hierarchical control of discrete-event systems. *Discrete Event Dynamic Systems* **6**, 241–306.